APPLICATION OF THE PML ABSORBING BOUNDARY CONDITION TO THE FDTD ANALYSIS OF MICROWAVE CIRCUITS

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Abstract - **The perfectly matched layer (PML) absorbing boundary condition is extended for the FDTD calculation of microwave circuits including dielectric boundaries. In addition a new source formulation is proposed, which becomes possible by the use of the PML. The theory is validated by the comparison of measured and calculated results for several microstrip components. It is shown, that the PML can be placed in the extreme nearfield of the structures under investigation without loss of accuracy.**

INTRODUCTION

The finite difference time domain method, which was introduced by Yee in 1966 [1], has found wide acceptance for the calculation of planar microwave circuits [2], [3], [4]. In [2] the FDTD was used to study several microstrip discontinuities including open-end, cross-junction, T-junction, gap and step-in-width. The flexibility of the FDTD with respect to the geometrical structures under investigation was proved by the analysis of more complex microstrip components like a radial stub and a spiral inductor [3]. Due to the introduction of a matched source [3] the direct separation of incident and reflected waves in the analysis of microwave circuits has become possible.

A basic element in the FDTD calculation of microwave circuits is the application of an absorbing boundary condition (ABC) to terminate the FDTD mesh. Widely used ABCs like Mur's [5] first and second order algorithm however offer several difficulties. First of all this ABCs cannot account for the dispersion on general transmission lines like for example a microstrip line. The phase velocity of an electromagnetic wave, which has to be absorbed, has to be constant in the frequency range of interest to avoid reflections by the ABC. Another drawback is the dependence of the ABC reflection coefficient on the angle of incidence of the electromagnetic wave.

The purpose of this paper is the application of a recently published ABC [6], which avoids the above mentioned difficulties, for the calculation of microwave circuits. In the next section the theory of this absorbing boundary condition is briefly summarised and the extension for general microwave circuits is given. In addition a new source formulation is proposed, which becomes possible by the use of the PML. Afterwards the theory is validated by the comparison of measured and calculated results for several microstrip components.

THEORY

1. Absorbing Boundary Condition

The perfectly matched layer (PML) was introduced by Berenger [6] and later extended to the three dimensional free-space case by Katz [7]. The following briefly summarises the characteristics of the PML, for more details see for example [7]. The PML consists of a non-physical lossy medium at the outer boundary of the FDTD space lattice backed by perfectly conducting (PEC) walls. Losses are introduced by specifying an electric conductivity σ and a magnetic conductivity σ^*. Due to the electric and magnetic losses an electromagnetic wave in the PML decays exponentially.

When the relation between the electric and magnetic conductivity is chosen properly [6], it can be shown that for the free-space case:

1. the wave impedance inside the PML equals the free-space wave impedance
2. the phase velocity inside the PML is the vacuum speed of light.

The application of the PML for the analysis of general microwave circuits needs some special treatment. First of all it is necessary to handle dielectrics, which extend to the perfectly matched layer. The second important aspect is the treatment of dielectric boundaries. For the FDTD, it is in common use to average the tangential permittivity and conductivity at the boundary between two different dielectric materials. A typical example is the tangential

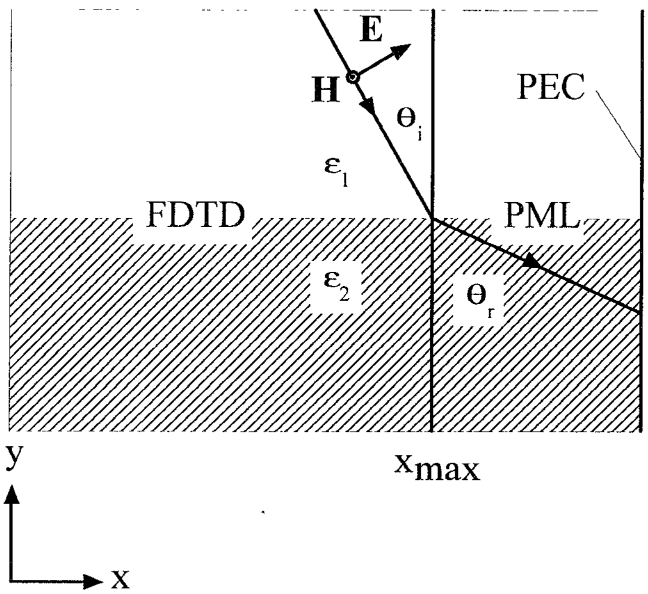


Fig. 1 Dielectric boundary extending to the PML region for the 2-D TE case.

permittivity ε_t at the dielectric-air interface of a microstrip circuit, which is calculated as $\varepsilon_t=(\varepsilon_0+\varepsilon_0\varepsilon_r)/2$ with ε_r the relative permittivity of the substrate [2].

Both problems mentioned above can be solved by reformulating the PML equations. The averaging of the tangential field between different dielectric materials can be taken into account by the permittivities ε_x, ε_y and ε_z. Using this approach, the differential equations implementing a PML become

$$\mu_0 \frac{\partial H_{xy}}{\partial t} + \sigma_{xy}^* H_{xy} = -\frac{\partial (E_{zx}+E_{zy})}{\partial y}, \tag{1}$$

$$\mu_0 \frac{\partial H_{xz}}{\partial t} + \sigma_{xz}^* H_{xz} = \frac{\partial (E_{yx}+E_{yz})}{\partial z}, \tag{2}$$

$$\mu_0 \frac{\partial H_{yz}}{\partial t} + \sigma_{yz}^* H_{yz} = -\frac{\partial (E_{xy}+E_{xz})}{\partial z}, \tag{3}$$

$$\mu_0 \frac{\partial H_{yx}}{\partial t} + \sigma_{yx}^* H_{yx} = \frac{\partial (E_{zx}+E_{zy})}{\partial x}, \tag{4}$$

$$\mu_0 \frac{\partial H_{zx}}{\partial t} + \sigma_{zx}^* H_{zx} = -\frac{\partial (E_{yx}+E_{yz})}{\partial x}, \tag{5}$$

$$\mu_0 \frac{\partial H_{zy}}{\partial t} + \sigma_{zy}^* H_{zy} = \frac{\partial (E_{xy}+E_{xz})}{\partial y}, \tag{6}$$

$$\varepsilon_x \frac{\partial E_{xy}}{\partial t} + \sigma_{xy} E_{xy} = \frac{\partial (H_{zx}+H_{zy})}{\partial y}, \tag{7}$$

$$\varepsilon_x \frac{\partial E_{xz}}{\partial t} + \sigma_{xz} E_{xz} = -\frac{\partial (H_{yx}+H_{yz})}{\partial z}, \tag{8}$$

$$\varepsilon_y \frac{\partial E_{yz}}{\partial t} + \sigma_{yz} E_{yz} = \frac{\partial (H_{xy}+H_{xz})}{\partial z}, \tag{9}$$

$$\varepsilon_y \frac{\partial E_{yx}}{\partial t} + \sigma_{yx} E_{yx} = -\frac{\partial (H_{zx}+H_{zy})}{\partial x}, \tag{10}$$

$$\varepsilon_z \frac{\partial E_{zx}}{\partial t} + \sigma_{zx} E_{zx} = \frac{\partial (H_{yx}+H_{yz})}{\partial x}, \tag{11}$$

$$\varepsilon_z \frac{\partial E_{zy}}{\partial t} + \sigma_{zy} E_{zy} = -\frac{\partial (H_{xy}+H_{xz})}{\partial y}. \tag{12}$$

The treatment of different dielectric materials, which extend to the PML region, requires the use of suitable electric and magnetic losses. Fig. 1 shows a simple example for the 2-D TE case. A plane wave, which is incident on a dielectric interface at an angle θ_i to the interface normal has to be absorbed by the PML at the boundary x_{max}. With $\sigma_{xy} = \sigma_{xz} = \sigma_x$ and $\sigma_{xy}^* = \sigma_{xz}^* = \sigma_x^*$ the necessary PML matching condition for reflectionless transmission between the FDTD and PML region is

$$\frac{\sigma_{x,i}}{\varepsilon_i} = \frac{\sigma_{x,i}^*}{\mu_0}, \quad i \in 1,2. \tag{13}$$

Inside the PML the electromagnetic wave decays exponentially in the x-direction [7] with a damping factor

$$\text{dielectric 1: } \frac{\sigma_{x,1}\sin\theta_i}{\varepsilon_1 c_1}, \text{ dielectric 2: } \frac{\sigma_{x,2}\sin\theta_r}{\varepsilon_2 c_2}, \tag{14}$$

where c denotes the phase velocity in the different materials. To allow for the same decay inside the PML for the different materials and the consideration of Snell's law $c_2 \sin\theta_i = c_1 \sin\theta_r$ lead to

$$\frac{\sigma_{x,1}}{\sigma_{x,2}} = \frac{\varepsilon_1}{\varepsilon_2}. \tag{15}$$

For a general multilayer structure this requirement is met, when the magnetic losses for all materials are the same.

2. Excitation Technique

Fig. 2 shows the source region for a microstrip circuit simulation. Excitation is done using a pulse with a Gaussian time dependence. The conventional way for exciting the line in Fig. 2 is to impose the vertical electric field in the excitation plane underneath the metalisation of the microstrip. Outside this rectangular region there exist an electric wall boundary condition in the excitation plane, when the Gaussian pulse is launched. This non-physical boundary condition leads to a dc tangential magnetic field in the vicinity of the excitation plane [2]. Therefore the minimum line length is restricted to a distance from the excitation plane, where the dc magnetic field has vanished.

An alternative efficient excitation technique is based on a current source formulation, which becomes possible by the use of the PML absorbing boundary condition. In this case the imposed field consists of a current density underneath the metalisation of the microstrip line as shown in

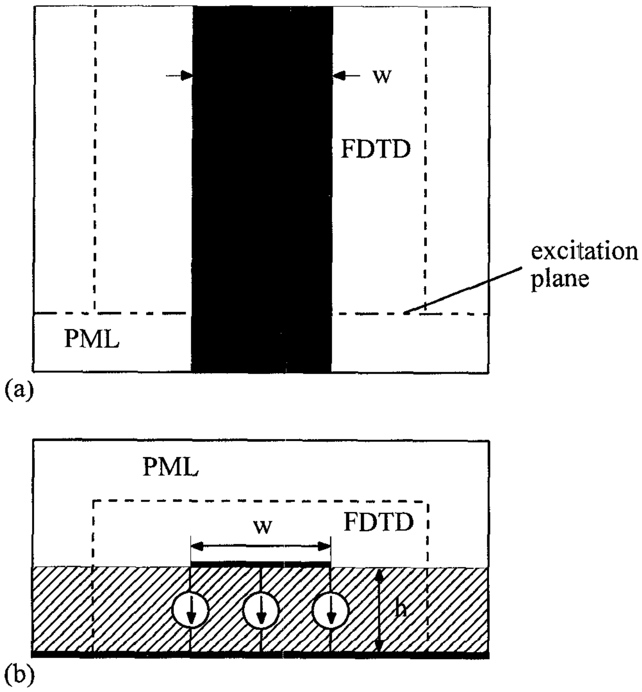


Fig. 2 Excitation of a microstrip line. (a) top view, (b) cross section.

Fig. 2b. Outside this rectangular region the electromagnetic field is determined in the conventional way from Maxwell's source free curl equations. This type of source offers several advantages compared with the electric wall source. The most important one is that there exist no dc magnetic fields in the vicinity of the excitation plane. Therefore the only restriction with respect to the necessary line length is to achieve the right modal characteristics of the line under investigation. The second advantage of a current source formulation is that this type of excitation is transparent to reflected waves in the general case of a microwave circuit including discontinuities.

RESULTS

To validate the FDTD including the extended PML absorbing boundary condition two microstrip circuit elements have been analysed. In both calculations the current source formulation was applied.

The first example is a meander line structure, which was first analysed by Wertgen [8] using a spectral domain technique. It consists of a section of closely coupled bends and transmission lines (Fig. 3a) on a Al_2O_3 substrate. The substrate height was h=635µm and the relative permittivity ε_r=9.978 (measured). For the FDTD analysis the structure was discretized with 104x23x11 elements. The PML thickness was 6 elements. In the direction perpendicular to the metalisation plane the distance between the metalisation and the absorbing boundary was only 5 space elements.

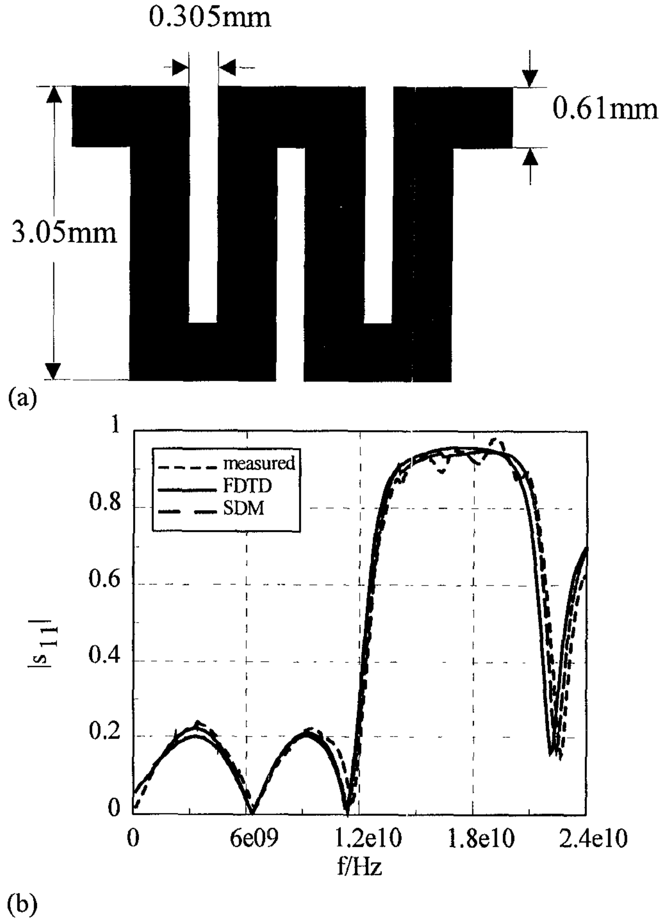
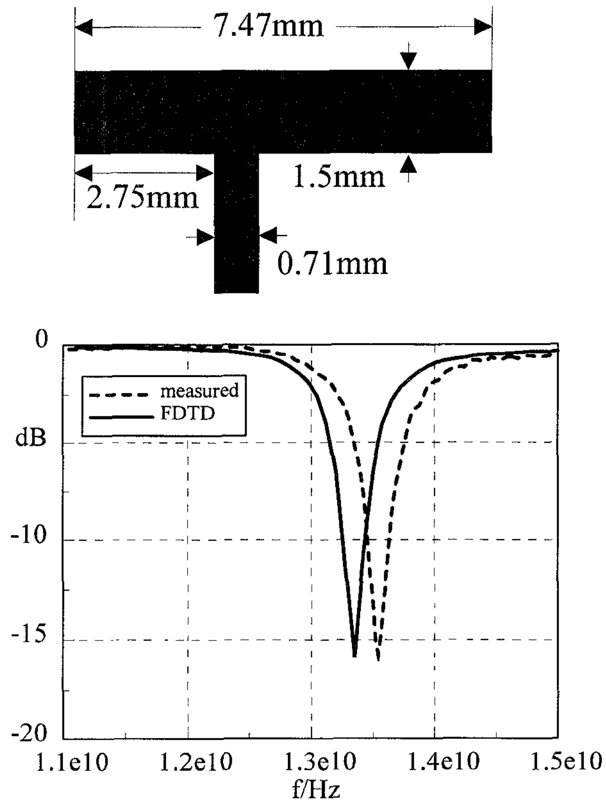


Fig. 3 Microstrip meander line. (a) geometry, (b) amplitude of s_{11}.

Fig. 3b shows the comparison of measurements, a spectral domain calculation [9] and the FDTD results for the amplitude of s_{11}. It can be seen, that the overall agreement between the numerical methods and the measurement is good. Only a small frequency offset between the full wave analyses and the measured results can be observed especially at the frequencies $f\cong$11GHz and $f\cong$22GHz. This is due to the fact, that there exist a small deviation in the geometry of the calculated and the real structure.

The next structure under investigation is a microstrip patch antenna. The microstrip antenna, which is shown in Fig. 4a, is fed by a microstrip line in the same plane. The substrate thickness is h=790µm and the relative permittivity ε_r=2.2. This structure was discretized with 91x36x12 cells and the PML thickness was 6 elements as in the first example. Fig. 4b shows a comparison between measured and calculated results for the amplitude of the



(a)

(b)

Fig. 4 Microstrip patch antenna. (a) geometry, (b) amplitude of r.

reflection coefficient. It can be seen, that there exist a frequency offset between the measured and calculated resonance frequency of the antenna of about 1.5%. This effect is well known from the literature [3], [10] and may due to the fact of growing dispersion of the FDTD space lattice for higher frequencies. Besides this the agreement between the measurement and the FDTD calculations is good especially for the bandwidth of the patch antenna.

CONCLUSION

In this contribution we have presented an extension of the PML absorbing boundary condition for the analysis of general microwave circuits. It has been shown that the PML can be placed in the extreme nearfield of the structures under investigation without loss of accuracy. Due to the superior performance of the PML compared to other absorbing boundary conditions the incorporation of a new current source formulation into the FDTD calculation was possible.

REFERENCES

[1] Yee, K. S.: Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media, *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302-307, 1966.

[2] Zhang, X. and Mei, K. K.: Time-Domain Finite Difference Approach to the Calculation of the Frequency-Dependent Characteristics of Microstrip Discontinuities, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, pp. 1775-1787, 1988.

[3] Wolff, I. and Rittweger, M.: Finite difference time-domain analysis of planar microwave circuits, *Archiv für Elektrotechnik*, vol. 74, pp. 189-201, 1991.

[4] Railton, C. J. and McGeehan, J. P.: Analysis of MMIC Components including the Effect of Finite Metalisation Thickness, *19th European Microwave Conf. Proc.*, London, 1989.

[5] Mur, G.: Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic-field equations, *IEEE Trans. Electromagn. Compat.*, vol. EMC-23, pp. 377-382, 1981.

[6] Berenger, J.-P.: A Perfectly Matched Layer for the Absorption of Electromagnetic Waves, *J. Computational Physics*, in press.

[7] Katz, D. S., Thiele, E. T. and Taflove, A.: Validation and Extension to Three Dimensions of the Berenger PML Absorbing Boundary Condition for FD-TD Meshes, *IEEE Microwave and Guided Wave Lett.*, pp. 268-270, 1994.

[8] Wertgen, W.: Elektrodynamische Analyse geometrisch komplexer (M)MIC-Strukturen mit effizienten numerischen Strategien, PhD Thesis, Duisburg University, FRG, 1989.

[9] Becks, T.: Elektrodynamische Simulation von passiven, dreidimensionalen Komponenten in (M)MIC-Schaltungen mit dem Spektralbereichsverfahren., PhD Thesis, Duisburg University, FRG, 1993.

[10] Kashiwa, T., Onishi, T. and Fukai, I.: Analysis of Microstrip Antennas on a Curved Surface Using the Conformal Grids FD-TD Method, *IEEE Trans. Antennas Propagat.*, pp. 423-427, 1994.